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On the construction of quintessential inflation models

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ABSTRACT: Attention has been recently drawn towards models in which inflation and quintessence schemes are unified. In such 'quintessential inflation' models, a unique scalar field is required to play both the role of the inflaton and of the late-time dynamical cosmological constant. We address the issue of the initial conditions for quintessence in this context and find that, in the two explicit examples provided, inflation can uniquely fix them to be in the allowed range for a present day tracking.

KEYWORDS: Dark Matter, Physics of the Early Universe.

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Contents

1. Introduction

The very last years have witnessed growing interest in cosmological models with $\Omega_m \sim 1/3$ and $\Omega_\Lambda \sim 2/3$, following the most recent observational data (see for example [1] and references therein). A very promising candidate for a dynamical cosmological constant $\Lambda(t)$ is a "quintessential" scalar field presently rolling down its potential [2], for which particle physics models have also been proposed [3]. The main motivation for constructing such dynamical schemes resides in the hope of weakening the fine tuning issue implied by the smallness of Λ.

In this respect, a very suitable class of models is provided by inverse power scalar potentials, $V \sim \phi^{-q}$ with $q > 0$, which admit attractor solutions in the scalar field dynamics [4, 5] characterized by a negative equation of state. The behaviour of these solutions is determined by the cosmological background and for this reason they have been denominated 'trackers' in the literature [5]. A good feature of these models is that for a very wide range of the initial conditions the scalar field will reach the tracking attractor before the present epoch [5]. This fact, together with the negative equation of state, makes the trackers feasible candidates for explaining the cosmological observation of a presently accelerating universe [6]. In this framework, indeed, it is possible to weaken the fine-tuning issue involved in choosing the initial conditions, even though the so-called 'cosmic coincidence' problem still has to be fixed by hand. While the initial value of the scalar energy density is irrelevant if the field comes on track before the present epoch, on the other hand the point at which the scalar and matter energy densities are of the same order depends on the mass scale in the potential. And this mass scale is fixed by requiring that $\Omega_\phi = \mathcal{O}(1)$ today.

It should be noted in passing that the requirement that the scalar field has already joined the attractor *today* is a crucial one. Indeed the field ϕ always passes though a 'freezing' phase with $w_\phi = -1$ before eventually reaching the tracker [5]. Although $w_\phi = -1$ is compatible with the present observational data, this value is indistinguishable from a 'true' cosmological constant and moreover it is not typical of the trackers. Any scalar field sitting in a non-vanishing minimum of its potential would give the same result, without any need of the dynamics to bring it there. In order to have the quintessence characteristic equation of state $-1 < w_\phi < 0$, we should then require that tha scalar field ϕ is already on track today.

As it will be discussed below, the range of initial conditions which allows ρ_ϕ to join the tracker before the present epoch is very wide. Nevertheless, it should be noticed that in principle we do not have any mechanism to prevent ρ_ϕ^{in} from being outside the desired interval. In this respect, an early universe mechanism which could uniquely fix it at the end of inflation is needed. In other words, if we find a way to naturally set ρ_ϕ^{in} in the range of values which allows for late time-tracking, we will be assured that the 'quintessence' field is a good candidate for the unknown component which presently accelerates the universe.

A promising way to address the problem of initial conditions for quintessence is the paradigm of "quintessential inflation" [7], also referred to as the "non oscillatory" [8] scheme. The basic idea is to study an inflaton potential $V(\phi)$ which, as it is typical in quintessence, goes to zero at infinity [7, 8, 9]. In this way it is possible to obtain a late time quintessential behaviour from the same scalar that in the early universe drives inflation. The hope is that the end of inflation could uniquely fix the initial conditions for the subsequent evolution of the scalar componet of the universe.

In ref. [7], a model with a potential which goes like $\sim \phi^4$ for $\phi < 0$ and like $\sim \phi^{-4}$ for $\phi > 0$ is studied in detail, and the authors use gravitational particle production for providing the entropy in the cosmological matter fields after the end of inflation. Although the shape of the potential for $\phi > 0$ is the same studied by Zatlev et al. in [5], they show that in this case the 'tracking' behaviour is not reached, because the initial conditions for the scalar energy density after inflation lay out of the phase space region that leads to joining the attractor before the present epoch. Anyway they succeed in matching the present cosmological data because the cosmological constant-like behaviour ($w_\phi = -1$) that they find for the scalar field is also also a viable option. The reason why the scalar does not reach the tracker is the fact that its energy density at the end of inflation is so low that it did not jet have enough time to move towards the attractor.

The model in [7] suffers from some problems with respect to the reheating mechanism that are extensively discussed in ref. [8]. In particular the authors of [8] propose to use the 'instant preheating' [10] mechanism instead of gravitational particle production for the post-inflationary reheating phase. The main foucus of that work, though, is not on the initial conditions for quintessence.

In this paper we address the issue of the initial conditions for quintessence in the context of the 'quintessential inflation' paradigm. The aim is two-fold. On one hand we will discuss under which conditions an inflaton potential can leave a residual vacuum energy on its tail, as already discussed in [7, 8]. On the other we will show that in some specific models it is possible to have a late time tracking (i.e. a well defined constrained behaviour of the scalar and negative but $\neq -1$ equation of state) of the residual inflaton energy density.

After briefly recalling the constraints which inflation and quintessence tracking models should separately meet, we will discuss to what extent they are compatible. We will then go on giving two specific examples, one in the context of first order inflation an the other in the hybrid case. In the first case we show that the 'escape point' from the tunneling naturally lies within the range which will produce a late time tracking. In the second, we re-examine the model proposed in [11] where a particle physics motivated potential was studied in order to produce inflation. We find that in that model, a late time quintessential behaviour is already built in and discuss under which conditions it meets the observational constraints.

1.1 Constructing workable models

Before starting the construction of any unified model of inflation and quintessence, it is necessary to establish the constraints to which it should be subject. This is not a trivial task, since both inflation and quintessence model building require very precise characteristics in order to be successful and we must check if these separate needs are compatible with each other.

Regarding *inflation*, there are four main points to be taken into account [12]:

1. If we want the universe to be accelerating, the equation of state of the inflaton ϕ

$$w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \tag{1.1}$$

must satisfy the inequality $w_\phi < -1/3$. This can be achieved if $\dot{\phi}^2 < V(\phi)$.

2. If inflation is to solve the flatness and horizon problems, a sufficient number of e-foldings should take place. This means that the ratio of the final to the initial value of the scale factor a must satisfy $a_f/a_i = \exp N$ with $N \gtrsim 50$.

3. The fact that the amplitude of scalar perturbations in the cosmic microwave background, as measured by COBE in 1992, is of order $\sim 10^{-5}$ constrains the normalization of the inflaton potential.

4. We must ensure that at the end of inflation sufficient reheating takes place. This is needed in order to produce the observed particle species in the universe. At the same time, one has also to check that gravitinos are not overproduced.

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This puts on the reheating temperature an upper limit which depends on the mass of the gravitinos and which is typically around $10^9 - -10^{12}$ GeV.[1]

For what concerns *quintessence*, the following requirements should be fulfilled (see [4, 5]):

1. In order for the scalar field modeling of the cosmological constant to be sufficiently general, we require that the post-inflationary shape of the potential is[2]

$$V(\phi) \sim \frac{\Lambda^{4+q}}{\phi^q}, \qquad q > 0\,. \tag{1.2}$$

   In this way we are guaranteed that for a very wide range of initial conditions (indeed between the present critical energy density and the background energy density at the beginning, $\rho^0_{cr} \leq \rho^{in}_\phi \leq \rho^{in}_{\rm B}$) the scalar field will be rapidly driven to a well-known "tracking" attractor behavior $\rho_{\rm TR}$:

$$\frac{\rho_{\rm TR}}{\rho_{\rm B}} \propto a^{\frac{6(w_{\rm B}+1)}{q+2}}, \tag{1.3}$$

   where $w_B = 0, 1/3$ during MD and RD respectively. The attractor is characterized by an equation of state that during MD is always negative: $w_{\rm TR} = (q\, w_{\rm B} - 2)/(q+2)$. There are two main qualitative ways through which this can be achieved (for more details see [5]). If the initial conditions for ϕ are such that $\rho^0_{cr} \leq \rho^{in}_\phi \leq \rho^{in}_{\rm TR}$ (*undershoot* case), then it will remain "frozen" until $\rho_\phi \sim \rho_{\rm TR}$ and then start to scale following eq. (1.3). If, instead, initially $\rho^{in}_{\rm TR} \leq \rho^{in}_\phi \leq \rho^{in}_{\rm B}$ (*overshoot* case) then ϕ will pass through a phase of kinetic energy domination before remaining frozen at $\rho_\phi < \rho_{\rm TR}$ and eventually join the attractor.

2. Secondly, we want the field ϕ to be already on track today and its present energy density to correspond to what observations report, i.e. $\Omega_\phi \simeq 2/3$. These two conditions translate to

$$V''(\phi) \simeq H^2 \qquad \text{and} \qquad V(\phi) \simeq \rho^0_{cr}\,, \tag{1.4}$$

   which together imply for the quintessence field $\phi \simeq M_p$ today. Moreover, eq. (1.4) provides a normalization for the mass scale Λ in the potential (1.2), giving

$$\Lambda \simeq \left(\rho^0_{cr}\, M_p^q\right)^{\frac{1}{4+q}} \simeq 10^{-\frac{123}{4+q}}\, M_p\,. \tag{1.5}$$

   This corresponds to choosing the desired tracker path to which the scalar will be attracted to.

[1] This bound refers to thermal production. However, recently attention has been paid to gravitinos production during preheating, suggesting that this mechanism could overcome the thermal one [13]. Since this depends on the precise form of the superpotential of the whole supergravity theory, we will not deal with it in the present work.

[2] This is the only class of potentials that admits an analytic "tracking" attractor solution. However, more general cases have been studied in ref. [5].

While it is straightforward to find potentials with the required early and late-time behavior, the subtle issue resides in successfully matching the exit conditions for the scalar field after inflation with the range of initial conditions allowed for the trackers. For example, the naive guess of using the potential $V = \Lambda^{4+q}\phi^{-q}$ for quintessential inflation is easily shown not to work. This is due to the fact that, with this potential, the slow-roll conditions imply $\phi \gtrsim M_p$ *during inflation*, while the request that the quintessence field is presently dominating the universe translates to $\phi \simeq M_p$ *today*.

The first scenario that we will discuss is first order inflation. In this context, if the potential $V(\phi)$ does not have an absolute minimum but goes to zero as ϕ runs to infinity, the exit conditions of the inflaton from the tunneling would set the starting point for the subsequent quintessential evolution of the same field ϕ. If instead hybrid inflation is considered, it is again possible to construct models in which the same field plays the role of the inflaton and of the late time dynamical cosmological constant. In this case, the critical value ϕ_c that makes inflation stop will determine the initial condition for the subsequent quintessential rolling.

2. The models

2.1 First-order quintessential inflation

In the original proposal of inflation [14], the scalar inflaton field ϕ which leads the expansion "sits" on a metastable minimum of its potential $V(\phi)$ during the whole process. Inflation eventually ends when bubbles of true vacuum nucleate through the barrier and subsequently expand and collide reheating the universe. A measure of the efficiency of the nucleation is given by the ratio

$$\varepsilon = \frac{\Gamma}{H^4} \tag{2.1}$$

between the tunneling rate Γ and the Hubble constant H.

As it was soon noticed [15], models where ε is constant in time cannot work, because one needs both (i) $\varepsilon \ll 1$ during inflation in order for the expansion to last enough to solve the flatness and the horizon problems and (ii) $\varepsilon \gtrsim 1$ to have an efficient nucleation. This puzzle is known as the "graceful exit problem". Many proposals have been suggested to solve this problem (see [16] for a review), based on the possibility of changing either H [17] or Γ [18] with time. This is commonly achieved by the use of an "auxiliary" scalar field ψ, which is also employed to fit the amplitude of scalar perturbations in the cosmic microwave background measured by COBE.[3] Without entering the details of this procedure, we will fix $\varepsilon = 1$ in the

[3] If this second field ψ is slowly rolling down its own potential $V(\psi)$, the amplitude of the density fluctuations is given by (see [19] for a review) $10^{-5} \simeq \delta\rho/\rho \simeq \frac{3H^3}{dV(\psi)/d\psi} \simeq 3(8\pi/3M_p^2)^{3/2}\frac{V(\phi)^{3/2}}{dV(\psi)/d\psi}$.

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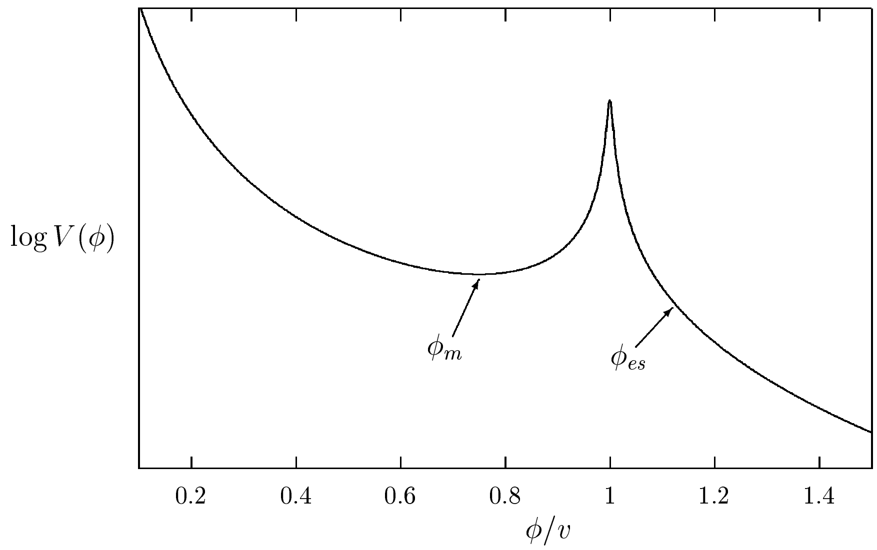

Figure 1: The bump in the potential of eq. (2.2), shown here with parameters $\alpha = 6$ and $\beta/v = 0.005$, allows for an early stage of inflation while the inflaton field ϕ sits in the relative minimum at ϕ_m. After ϕ has tunneled out at ϕ_{es}, the quintessential phase starts with the scalar rolling down the slope $\sim \phi^{-\alpha-2}$ until today.

toy model below, *assuming* that also in our case some auxiliary field (or some other mechanism) can be invoked in this regard.[4]

In the model that we propose, the scalar field ϕ has a potential (see figure 1)

$$V(\phi) = \frac{\Lambda^{\alpha+6}}{\phi^\alpha \left[(\phi - v)^2 + \beta^2\right]}\,, \qquad \text{with } \frac{\beta}{v} \ll 1\,, \tag{2.2}$$

where Λ, β and v are constants of mass dimension one. Eq. (2.2) has a barrier at $\phi \sim v$, after a metastable minimum in $\phi \sim v\alpha/(\alpha+2) \equiv \phi_m$, while it behaves like $\sim \Lambda^{\alpha+6}/\phi^{\alpha+2}$ for $\phi \gg v$.

The parameter Λ is constrained by quintessence (see eqs. (1.4)–(1.5)):

$$\Lambda \simeq 10^{-\frac{123}{\alpha+6}}\, M_p \simeq 10^{\frac{19\,\alpha-9}{\alpha+6}}\ \mathrm{GeV}\ . \tag{2.3}$$

In this way we ensure that the residual vacuum energy after inflation does not overclose the universe and that at the same time it is not presently negligibly small.

Inflation, instead, requires that most of the energy density $V(\phi_m)$ which dominates the accelerated expansion is transferred, after the end of inflation, into a thermal bath of temperature $T_{rh} \equiv 10^{9+\gamma}\,\mathrm{GeV}$ and this fixes the scale v in the potential[5]

$$v \simeq (\alpha+2)\left(\frac{10^{-83-4\,\gamma}}{140\,\alpha^\alpha}\right)^{\frac{1}{\alpha+2}} M_p \simeq (\alpha+2)\left(\frac{10^{19\alpha-45-4\gamma}}{140\,\alpha^\alpha}\right)^{\frac{1}{\alpha+2}}\ \mathrm{GeV}\,. \tag{2.4}$$

[4]Since a late time quintessential behaviour obvoiusly cannot affect the exit from inflation, the solutions proposed so far (see for example [17, 18]) may be assumed to work also in the present case.

[5]For the degrees of freedom of the Standard Model $V(\phi_m) \simeq \rho_{rad} \simeq 35\,T_{rh}^4$.

The ratio β/v is fixed below by the condition $\varepsilon = 1$, so that the only free parameters of the model are the exponent α and the reheating temperature parametrized by γ.

Although the potential (2.2) does not have two minima (being the lower one at infinity), the problem can be analytically approached in the so called "thin wall limit" [20] as in the case in which the two minima are present. This limit applies when the barrier is much higher than the difference between the two minima. This is our case, since

$$\frac{V(v)}{V(\phi_m)} = \mathrm{O}\left(\frac{v}{\beta}\right)^2 \gg 1\,. \tag{2.5}$$

In order to get the decay rate Γ (see [20] for details) one has to integrate the equation of motion associated to the potential (2.2) and select the solution $\phi(x)$ which minimizes the euclidean action S_E of the system. This solution is O(4) symmetric (in the whole euclidean space) and approaches $\phi = \phi_m$ at $x \equiv |\mathbf{x}| = \infty$. The value $\phi(x = 0) \equiv \phi_{\rm es}$ is called the "escape point" and corresponds to the point at which the field ϕ tunnels out and starts rolling under its classical equation of motion.

If the thin-wall limit holds, the solution is $\phi \simeq \phi_{\rm es}$ for an interval $0 < x < R$, and then $\phi \simeq \phi_m$ for $x > R$. We physically interpret it as a bubble with radius R of (nearly) true vacuum within separated by a thin wall from the false vacuum without. Continued to Minkowski space, the bubble appears to expand with a speed which asimptotically approaches the speed of light. The universe can then be reheated by the particle production that occurs during the subsequent phase of collision of the bubbles recovering from the tunneling. The dynamics of this process in the present model is exactly analogous to the one which occurs in the usual case (when the minimum of $V(\phi)$ is at a finite value of ϕ) and is extensively discussed in ref. [21].

Following [20], the euclidean action and the initial radius of the bubbles are given by

$$\begin{aligned} S_E &\sim -\frac{27\,\pi^2}{2}\cdot\frac{S_1^4}{V(\phi_m)^3} \\ R &\simeq 3\frac{S_1}{V(\phi_m)}\,, \end{aligned} \tag{2.6}$$

where

$$S_1 = \int_{\phi_m}^{\infty} d\phi\,[2\,(V(\phi) - V(\phi_m))]^{1/2}\,. \tag{2.7}$$

In our case S_1 can be calculated analytically for any value of α in the potential (2.2) without any approximation, but a more readable and accurate enough estimate is given by

$$S_1 \simeq 2\int_{\phi_m}^{v} d\phi\,\sqrt{2}\left[\frac{\Lambda^{\alpha+6}}{v^\alpha}\,\frac{1}{(\phi - v)^2 + \beta^2}\right]^{1/2} = \frac{2\sqrt{2}\,\Lambda^{(\alpha+6)/2}}{v^{\alpha/2}}\ln\left[\frac{4}{\alpha+2}\,\frac{v}{\beta}\right]. \tag{2.8}$$

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γ	α	β/v	R_{an} $[\mathrm{GeV}^{-1}]$	R_{num} $[\mathrm{GeV}^{-1}]$	ϕ_{es}/v
1	8	$7.75 \cdot 10^{-3}$	$1.06 \cdot 10^{-10}$	$0.98 \cdot 10^{-10}$	1.30
2	10	$9.72 \cdot 10^{-3}$	$1.03 \cdot 10^{-11}$	$0.94 \cdot 10^{-11}$	1.23
3	8	$4.74 \cdot 10^{-4}$	$9.88 \cdot 10^{-13}$	$9.07 \cdot 10^{-13}$	1.25

Table 1: Comparison between the analytical and numerical results for the radius of the tunneling bubble, for some values of the parameters of the model. In the last column, the escape point is given in units of v.

The tunneling rate of ϕ is

$$\Gamma = A\, e^{-S_E} \tag{2.9}$$

where A is a parameter with mass dimension 4 of order $V(\phi_m)$.

From this equation, the condition

$$\varepsilon = \frac{\Gamma}{H^4} = \left(\frac{3}{8\pi}\right)^2 \frac{M_p^4}{V(\phi_m)}\, e^{-S_E} = 1 \tag{2.10}$$

is obtained for $S_E \simeq 84 - 9\,\gamma$, that is if the ratio β/v satisfies

$$\ln\left[\frac{4}{\alpha+2}\,\frac{v}{\beta}\right] = \left(\frac{84 - 9\,\gamma}{5.5\cdot 10^5} 140^{\frac{\alpha+6}{\alpha+2}}\right)^{1/4} \left(\frac{\alpha+2}{\alpha^{\frac{\alpha}{\alpha+2}}}\right)^{\alpha/2} \left(10^{\alpha\,(\gamma-10)+63+6\,\gamma}\right)^{1/(\alpha+2)}. \tag{2.11}$$

Moreover, with this condition we also have

$$R = 0.36\,(84 - 9\,\gamma)^{1/4}\; 10^{-9-\gamma}\,\mathrm{GeV}^{-1} \tag{2.12}$$

as the analytical estimate for the initial radius of the bubbles.[6]

Going on with the analysis, we specify to some particular values of the parameters. As anticipated in section 1.1, we impose to the reheating temperature the upper bound $T_{rh} \leq 10^{12}\,\mathrm{GeV}$, that is $\gamma \leq 3$. We see from eq. (2.11) that, for any fixed value of γ, it is always possible to obtain $\varepsilon = 1$ with arbitrarily low β/v, just allowing α to be large enough. However, for phenomenological reasons (see below) we restrict ourselves to $\alpha \lesssim 10$ and list[7] in table 1 the cases for which $\beta/v \ll 1$.

We studied the tunneling also numerically and the solutions that we found are in good accordance with the previous semi-quantitative analysis. In particular, their shape is that of an instanton which interpolates between the initial value ϕ_{es} and the final one ϕ_m. The jump between the two values occurs at $x \equiv R_{num}$, very close to the analytical estimate R_{an} given by eq. (2.12), as can be checked in table 1.

[6]In all this analysis we have not considered the gravitational corrections on the decay of the metastable vacuum. However, since they are of order $(RH)^2$ [22], their contribution is completely negligible in all the cases of our interest.

[7]In order to avoid excessive fine tuning, we have not listed the cases for which $\beta/v < 10^{-5}$. However this is a somewhat arbitrary limit and nothing prevents from considering smaller values.

At this stage, it is easily understood that the initial conditions for quintessence are entirely determined by the tunneling and not given arbitrarily. In particular, if the present model is considered, when ϕ tunnels out at the escape point ϕ_{es} and starts rolling down the $V \sim \phi^{-\alpha-2}$ potential, it has an energy density given by

$$V(\phi_{es}) \simeq V(\phi_m)\frac{4\,\alpha^\alpha}{(\alpha+2)^{\alpha+2}}\frac{(v/\phi_{es})^\alpha}{(\phi_{es}/v-1)^2} \simeq \rho_{rad}\,\frac{4\,\alpha^\alpha}{(\alpha+2)^{\alpha+2}}\frac{(v/\phi_{es})^{\alpha+2}}{(1-v/\phi_{es})^2} \tag{2.13}$$

which is typically about 1% of the thermal one and which can be computed as a function of α and of the reheating temperature substituting the values of table 1 in the last expression. Note also that the field ϕ at the beginning of the "quintessential" regime is of the order of v and, from eq. (2.4), can be easily estimated to be $\ll M_p$ (actually it tends to M_p when $\alpha \to \infty$).

These initial conditions naturally lay within the allowed range for quintessence and correspond to the *overshoot* case mentioned in section 1.1. The field ϕ will then rapidly run to large values and its energy density will consequently drop down well below the tracker, as discussed in ref. [5]. Then, after a "freezing" phase of almost zero kinetic energy, it will eventually join the tracker path at more recent times. As a function of the exponent α in the potential (2.2), the equation of state of the scalar ϕ on the tracker is

$$w_\phi = -\frac{2}{\alpha+4}\,. \tag{2.14}$$

However it should be remembered that the present value of the equation of state is lower than the attractor value. When the scalar energy density ceases to be subdominant with respect to the matter one, the approximation in which the attractor was derived does not hold anymore [5]. The scalar then leaves the tracking path as soon as its energy density is comparable to that of matter and rapidly tends towards a cosmological constant-like behaviour with $w_\phi = -1$. For a present ratio $\Omega_\phi/\Omega_m \simeq 2$ we should restrict to $\alpha \lesssim 10$ to be compatible with the present data [6].

2.2 Hybrid quintessential inflation

The model that we will consider next was proposed in [11] and involves a scalar potential arising from dynamical supersymmetry breaking, of the form $V = V_{\rm SUSY} + V_{\cancel{\rm SUSY}}$, with

$$V_{\rm SUSY} = M^4\left|1-\lambda\frac{\chi^2\phi^2}{M^4}\right|^2 + \frac{\Lambda^{4+p}}{\phi^p}\,, \qquad V_{\cancel{\rm SUSY}} = \frac{1}{2}\beta M^2\chi^2\,. \tag{2.15}$$

As extensively discussed in [11], this potential can easily accomodate an early inflationary stage of the hybrid [23] type.[8] We find that, quite surprisingly, this model

[8] The presence of a SUSY-breaking mass term for the scalar ϕ is cosmologically excluded if we require a late time quintessential behaviour, since it would induce a minimum in the ϕ-direction of the potential.

has already incorporated a late-time quintessential phase and this leads to important consequences in addition to those discussed in [11]. The interesting point is that this class of potentials is the first example which allows for discussing quintessential inflation in a particle physics context. While this issue is discussed both in the purely inflationary (see the third reference in [12]) or purely quintessential [3] cases, it is still missing in the 'quintessential inflation' scheme. In the following we will then address this problem with the potential (2.15).

For $\phi < \phi_c = \sqrt{\beta M^2/2\lambda}$ the minimum of the potential is at $\chi = 0$ and inflation can occur if the term M^4 in (2.15) dominates. When ϕ rolls down to $\phi > \phi_c$ inflation is ended by instability in the χ direction, as typically occurs in hybrid models. In this case, however, the VEV of the scalar potential V does not almost instantaneously settle to zero but vanishes only after ϕ has run to infinity. This feature is very welcome if we want a quintessential component to be present in the subsequent evolution of the universe. In what follows we study for which range of the parameters this model can fulfill the double aim of accounting for both the inflationary and quintessential stages of our universe.

For $\chi = 0$ and $\phi < \phi_c$, the potential can be rewritten as

$$V = M^4 \left[1 + \alpha \left(\frac{M_p}{\phi} \right)^p \right] \tag{2.16}$$

with

$$\alpha \equiv \frac{\Lambda^{p+4}}{M_p^p \, M^4} \, . \tag{2.17}$$

We will see below that stringent upper limits apply to α for the model to fit observations. For the moment we only ask α to be small enough so that the constant term dominates eq. (2.16), leading to a first inflationary stage. This is naturally achieved if we require that the term Λ^{4+p}/ϕ^p in eq. (2.15) leads to a present energy density that does not exceed the critical one (see section 1.1), that is if $\Lambda \leq \Lambda_c$ with

$$\Lambda_c^{p+4} = 10^{-123} \, M_p^{p+4} \, . \tag{2.18}$$

To estimate the starting point of inflation we consider the slow roll parameters

$$\varepsilon \equiv \frac{M_p^2}{4\,\pi} \left(\frac{H'(\phi)}{H(\phi)} \right)^2 \simeq \frac{M_p^2}{16\,\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 = \left(\frac{\phi_0}{\phi} \right)^{2(p+1)} \tag{2.19}$$

$$\begin{aligned} \eta \equiv \frac{M_p^2}{4\,\pi} \left(\frac{H''(\phi)}{H(\phi)} \right) &\simeq -\frac{M_p^2}{8\,\pi} \left[\frac{V''(\phi)}{V(\phi)} - \frac{1}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \right] \\ &= \left(\frac{p+1}{2\sqrt{\pi}} \right) \left(\frac{\phi_0}{\phi} \right)^{p+1} \frac{M_p}{\phi} + \left(\frac{\phi_0}{\phi} \right)^{2\,(p+1)} \end{aligned} \tag{2.20}$$

where a prime denotes differentiation w.r.t. ϕ and

$$\phi_0 = \left(\frac{p}{4\,\sqrt{\pi}} \, \alpha \right)^{1/(p+1)} M_p \, . \tag{2.21}$$

Since slow roll requires $\varepsilon, \eta \ll 1$, the accelerated expansion occurs for $\phi_0 \ll \phi \leq \phi_c$.

When inflation ends at $\phi = \phi_c$, the second scalar χ leaves zero (that becomes an unstable maximum of V) and oscillates about one of the two new ϕ-dependent minima that form at

$$\chi^2_{\rm MIN} = \frac{2M^2}{\beta}\frac{\phi_c^2}{\phi^2}\left(1 - \frac{\phi_c^2}{\phi^2}\right) . \tag{2.22}$$

If we suppose that the scalar χ is coupled to the matter fields via terms like $h\chi^2\varphi^2$, its coherent oscillations would result in sufficient reheating of the universe by its decay products. The scalar χ eventually settles to $\chi_{\rm MIN}$ and the reheating temperature is typically of order $\sim M$. After the reheating phase is completed the potential is given by

$$V(\chi_{\rm MIN}, \phi) = M^4\frac{\phi_c^2}{\phi^2} + \frac{\Lambda^{4+p}}{\phi^p} . \tag{2.23}$$

It should be noted that, taking into account the bound (2.28) on Λ given below, it turns out that at late times ($\phi \gg \phi_c$) the term that will dominate in the above formula is the first one. That means that the tracking behavoiur will not be induced, as might be expected, by the explicit inverse power term present in the potential, but will instead come from the effective contribution $\sim \phi_c^2/\phi^2$ originated by the minimum of the second scalar χ.

The first term in eq. (2.23) constrains the ratio of ϕ_c to M, since we need that today $V(\chi_{\rm MIN}, \phi \simeq M_p) \simeq \rho^0_{cr}$. This translates to

$$\frac{\phi_c}{M} = \sqrt{\frac{\beta}{2\,\lambda}} \simeq \left(\frac{100\,{\rm GeV}}{M}\right)^3 \cdot 10^{-10} . \tag{2.24}$$

The smallness of ϕ_c constrains the other parameters of the model, once the two main requirements of inflation, namely that it lasts for enough e-folds and that it gives the spectrum of fluctuations observed by COBE, are taken into account. The number of e-folds of inflation, when $\eta \gg \varepsilon$ is given by [11]

$$N_{\rm tot} = \frac{2\sqrt{\pi}}{M_p}\int_{\phi_{in}}^{\phi_c}\frac{d\,\phi'}{\sqrt{\varepsilon\,(\phi')}} \simeq \frac{8\pi}{p(p+2)}\frac{1}{\alpha}\left(\frac{\phi_c}{M_p}\right)^{p+2} . \tag{2.25}$$

Since at least 50 or 60 e-folds are needed to solve the horizon and flatness problems, we impose $N_{\rm tot} \geq 50$. For what concerns the fluctuations, instead, we require that the curvature power spectrum[9] [11]

$$P_R^{1/2} \equiv \frac{1}{\sqrt{\pi}}\frac{H\,(\phi_{50})}{M_p\,\sqrt{\varepsilon\,(\phi_{50})}} \simeq \frac{p+2}{\pi}\sqrt{\frac{2\,\pi}{3}}\left(\frac{M^2}{M_p\,\phi_c}\right)\,N_{\rm tot}\,\left(1 - \frac{50}{N_{\rm tot}}\right)^{(p+1)/(p+2)} \tag{2.26}$$

[9] ϕ_{50} is the value of the inflaton field 50 e-folds before the end of inflation.

matches the COBE normalization [24] $P_R^{1/2} = 5 \cdot 10^{-5}$, with spectral index [11]

$$n - 1 \equiv \frac{d\,\log(P_R)}{d\,\log k} = -4\varepsilon + 2\eta \simeq \left(\frac{p+1}{p+2}\right) \frac{2}{N_{\rm tot}(1 - 50/N_{\rm tot})} \,. \tag{2.27}$$

The spectrum turns out to be blue ($n > 1$), but for $N_{\rm tot} > 50$ it quickly approaches scale invariance, $n \simeq 1$. The present limit $|n - 1| < 0.2$ [12] translates to $N_{\rm tot} \gtrsim 60$.

Substituting eqs. (2.17) and (2.24) into eq. (2.25), we obtain the following upper bound on the mass scale[10] Λ:

$$\left(\frac{\Lambda}{\Lambda_c}\right)^{p+4} \simeq \frac{10^{-27p}}{p(p+2)2^{p+1}} \left(\frac{100\,{\rm GeV}}{M}\right)^{2p} \left(\frac{50}{N_{\rm tot}}\right) . \tag{2.28}$$

Finally, from eq. (2.26) we get the rough estimate $\frac{M^2}{M_p\,\phi_c}\,N_{\rm tot} \sim 10^{-5}$, which translates into

$$M \simeq 100\,{\rm GeV} \left(\frac{50}{N_{\rm tot}}\right)^{1/4} . \tag{2.29}$$

We thus understand that a sufficient amount of e-folds can be achieved only for a quite low reheating temperature (remember $T_{rh} \simeq M$). Anyhow, a low T_{rh} is also preferred since it weakens the upper bounds on ϕ_c and on Λ given by eqs. (2.24) and (2.28).

As an example of the orders of magnitude involved, for $M \simeq 50\,{\rm GeV}$ we get $N_{\rm tot} \simeq 800$, $\phi_c \simeq 22\,{\rm eV}$, and Λ in the range $0.016\,{\rm eV}(p = 2) \div 5\,{\rm eV}(p = 50)$. The last mass scale is not very natural in a supersymmetric context, where one customarily expects values $\gtrsim$ GeV. However, the amount of fine tuning involved in Λ from eq. (2.28) is milder than in the case of the cosmological constant, where the mass scale is more than 30 orders of magnitude smaller than the "natural" value.

The last important point in our discussion is the "quintessential" evolution of ϕ after the reheating phase. As already noticed, the bound on Λ given by eq. (2.28) forces the second term in eq. (2.23) to be completely negligible during this last phase. Despite its shape is exactly the one required for the trackers, the only role it plays in this model is to drive ϕ towards ϕ_c during inflation. The term which dominates the potential (2.23) at late times comes instead from the dynamics of the field χ and the tracking behavior is guaranteed from the fact that it involves a negative power of the inflaton ϕ as well.

The initial conditions of this "quintessential" phase are fixed by the value ϕ^* of the field ϕ after reheating, when $\chi = \chi_{\rm MIN}$ and eq. (2.23) starts holding. The precise value of ϕ^* depends on the details of the physics which governs the reheating, but it is reasonable to assume that it will not be much larger than ϕ_c. If this is the case, the initial energy of the quintessential field ϕ will be somewhat smaller (but not too smaller) than the one stored in the thermal background and, as in the previous model, we are again in the "overshoot" case.

The attractor equation of state for a potential $V \sim \phi^{-2}$ is simply $w_\phi = -1/2$, well within the observational bound [6].

[10] Inserting this value for Λ in eq. (2.21) we can check that, consistently, $\phi_0 \ll \phi_c$.

3. Conclusions

In this paper we have discussed two possible schemes in which inflation and quintessence are unified. In both cases it is the same field which at the same time plays the role of the inflaton and of the quintessence scalar. In this way we succeded to uniquely fix the initial conditions for quintessence from the end of inflation and have found that they are compatible with a late-time tracking.

In one example we studied first-order inflation with a potential going to zero at infinity like $\phi^{-\alpha}$. A bump in the potential at $\phi \ll M_p$ allows for an early stage of inflation while the scalar field gets "hung up" in the metastable vacuum of the theory. Nucleation of bubbles of true vacuum through the potential barrier sets the end of the accelerated expansion and starts the reheating phase. As it is well known, this scenario suffers from the so-called "graceful exit problem", but we briefly commented on possible ways out where (thanks to some auxiliary scalar field) the ratio of the tunneling rate to the Hubble volume, Γ/H^4, varies with time. After the reheating process is completed, the quintessential rolling of the scalar ϕ starts and its initial conditions (uniquely fixed by the end of inflation) are naturally within the range which leads to a tracking behavior in recent times.

As an alternative, we considered the model of hybrid inflation which, motivated by dynamical supersymmetry breaking, was proposed by the authors of [11]. We showed that it naturally includes a late-time quintessential behavior. This result is very interesting since it is the first time that the quintessential inflation scheme is discussed in a particle physics motivated context. As typical of hybrid schemes, the potential is dominated at early times (that is until the inflaton field is smaller than a critical value ϕ_c) by a constant term and inflation takes place. Eventually the inflaton rolls above ϕ_c, rendering unstable the second scalar of the model, χ. This field starts oscillating about its minimum (whose position is determined by ϕ) and in this stage the universe is reheated. After χ has settled to the minimum, the inflaton continues its slow roll down the "residual" potential which goes to zero at infinity like ϕ^{-2}, thus allowing for a quintessential tracking solution. Also in this case the initial conditions for the quintessential part of the model do not have to be set by hand, but depend uniquely on the value of the inflaton field at the end of reheating.

In our analysis we have shown that in both models inflation and quintessence can be accounted for within a relatively wide range of the parameters of the potential. Some differences exist though. The first one concerns the exponent p characterizing the late time shape of the potential $V(\phi) \sim \phi^{-p}$ which leads the quintessential part of the models. While in the first case the exponent can be read directly from the potential, in the other the leading contribution comes from the dynamics of the second scalar χ which gives $V \sim \phi^{-2}$. The explicit term proportional to ϕ^{-p} in the potential of this last model is just needed for driving the inflaton towards ϕ_c during

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inflation and then is completly negligible. Moreover, contrary to the first scheme, the dependence of the physical quantities on p is in this case very mild and hardly distinguishable by observations.

Another important difference between the two analyses is that in the first-order model we allowed for relatively high reheating temperatures, while in the second case we obtained $T_{rh} \lesssim 100\,\mathrm{GeV}$. A low reheating temperature can still be accommodated in a successful cosmology, since the only model independent limit is $T_{rh} \gtrsim$ GeV in order to produce protons and neutrons. Moreover such small values could avoid the restoration of symmetries which in many extensions of the Standard Model would produce a number of unwanted topological defects.

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